# "*Constructive Skepticism*" Volume 3 – Notebook #I: Model Risk

## Chapter 5: The "*Roughness*", Complexity, and Randomness of the Preference Problem

As we continue to develop "*Tools, Checklists & Processes*" to evaluate Model Risk in order to find decent models for retirement planning, the previous chapter focused on the "*Statistical illusions*" from Statistical Machinery that apply to the Measurement Problem ("*Statistical Meaning*"), articulated the nature of the model risk of all model risks [*Confusing a part for the whole*], and closed by connecting the problems to the foundational "*Axioms, Assumptions & Hypotheses*" that pertain to most quantitative models.

While the Measurement Problem introduced the idea of the Random Variable, the Preference Problem introduces the idea of the Stochastic Process. Useful models for retirement planning combine the probability distributions of random variables with mathematical functions that describe growth formulas in the form of Stochastic Processes. Modelers then use the resulting "*Growth Dynamics*" to realize sample trajectories in order to calculate summary statistics such as averages and growth rates. Finally, these models can chain series of such combinations and calculations, including functional transformations of the starting growth formula.

This chapter shows that model building becomes a series of choices of random variables, probability distributions, growth formulas, stochastic processes, and realized trajectories to calculate averages, and growth rates in order to characterize alternatives for decision making. It also illustrates how intuition leaps over the steps of such "*Rational*" analysis in order to make repeated, timely, and accurate decisions. The Preference Problem shows that the "*Brain*" takes short-cuts across the steps of "*Rational*" models to quickly find the real-life relevant and existential features. For instance, human common sense intuits the difference between sample statistics, and population parameters.

### ***Gerd Gigerenzer***

***Gerd Gigerenzer*** showed that individuals have intuitions that connect to a better fine-grain understanding of "*Randomness*" than the form of "*Rational*" thinking used in the ***Heuristics & Bias Program***. Further, ***Gigerenzer*** showed the limits of the "*Rational*" theory of Psychological Preferences, especially in its sub-fields – such as Behavioral Economics, and Behavioral Finance – that make "*Rationality*" the benchmark for modeling, and measuring "*Bias*".

In his 2023 book titled *"The Intelligence of Intuition"*, ***Gigerenzer*** shows how research designs behind key results of the ***Heuristics & Bias Program*** conflate "*Logical Equivalence*" with "*Informational Equivalence*" by failing to take "*Task Environments*" into consideration. Using the example of intuitive "*Perceptions*" of "*Randomness*" - a central finding for the validation of recommendations such as "*Nudging*" from ***Behavioral Economics***, ***Behavioral Finance***, and their overall umbrella of the ***Heuristics & Bias Program - Gigerenzer*** shows how research designs that do not make the distinction between "*Population Statistics*" and "*Sample Statistics*" create results that do not work in real-life.

Research designs of the ***Heuristics & Bias Program*** that use flipping a coin several times, and recording the patterns of Heads ("H") & Tails ("T") in order to test human intuitions about "*Randomness*", use "*Population Statistics*" hen the number of throws ("n") equals the length of the observed sequence of throws ("k").

In this specific & unique case where n = 3, and k = 3, we observe eight cases ranging from (H, H, H) to (T, T, T). Each case has the same probability of 1/8, and the sequence (H, H, H) has the same probability as the sequence (H, H, T).

Researchers from the ***Heuristics & Bias Program*** use such results from "*Population Statistics*" as evidence of an irrational bias when human subjects express their intuitive opinion that the sequence (H, H, T) is more likely to occur than the sequence (H, H, H). Note how this assertion of irrationality extends the range of applicability of the results from a specific case of "*Population Statistics*" to a general statement about "*Sample Statistics*".

Research designs of the "*Fast & Frugal*" ***Heuristics Program*** that use flipping a coin several time, and recording the patterns of Heads ("H") & Tails ("T") in order to test human intuitions about "*Randomness*", use "*Sample Statistics*" when the number of throws ("n") becomes greater than the length of the observed sequence of throws ("k").

In the many cases where k < n < Infinity in general, and using n = 4 and k = 3 as an example in particular, we observe sixteen cases ranging from (H, H, H, H) to (T, T, T, T). Each case has the same probability of 1/16. However, the observed sequence (H, H, H) occurs in three of these cases while the observed sequence (H, H, T) occurs in four of these cases. This means that the observed, sample sequence (H, H, H) has a relative frequency of 3/16 = .19, and the observed, sample sequence (H, H, T) has a relative frequency of 4/16 = .25.

Researchers from the "*Fast & Frugal*" ***Heuristic Program*** use such results from "*Sample Statistics*" as evidence that human intuition is correct when human subjects express their intuitive opinion that the sequence (H, H T) is more likely to occur than the sequence (H, H, H). Note how this assertion of "*Ecological Rationality*" matches the applicability of the results to "*Task Environments*" defined by "*Sample Statistics*".

This distinction highlights a core problem with the Heuristics & Bias Program because humans make decisions with samples of "*Observations*" that require the use of "*Sample Statistics*" as opposed to the use of "*Population Statistics*". ***Gigerenzer*** documents other cases to show how the ***Heuristics & Bias Program*** has a bias in seeing individual biases in everything, and calls it the "*Bias Bias*". This "*Bias Bias*" explains the lack of reproducibility of famous results that, to the surprise of many, include the expected effects of Framing, the irrationality of intuitive “*Perceptions*” of “*Randomness*” [as illustrated in the above example], and the “*Hot Hand*” Fallacy.

Yet, the research findings of the ***Heuristics & Bias Program*** dominate the thinking of public and private institutions, perhaps in part - and not just a small part as ***Gigerenzer*** observes - because they line up easily into some of the historical battle lines of Psychology, including the battle between institutional paternalism vs. individuals capable of giving informed refusal as well as informed consent. ***Gigerenzer*** pointed out that foundational work such as the work of ***Jean Piaget***, and others concluded that by age 12 children demonstrated good individual judgment and intuitions about chance, frequency, and randomness, up to the point of a 1967 review of 110 such papers by ***Cameron Peterson and Lee Roy Beach***, and aptly titled “*Man as an Intuitive Statistician*”.

However, starting in 1973, ***Amos Tversky & Daniel Kahneman*** wrote a paper, “*Judgment under Uncertainty: Heuristics & Biases*” that summarized 4 of their own published papers, and took the field of Psychology in the opposite direction. It is this 1973 paper by ***Tversky & Kahneman*** that set the direction for the ***Heuristics & Bias Program***. This direction seeks to prove that people do not have good individual judgment and intuitions about chance, frequency, and randomness, and thus must be guided by their betters with interventions that range from surreptitious “*Nudging*”, to the “*In-Your-Face*” exercise of power.

When it comes to retirement planning, ***Gigerenzer*** shows a variety of reasons why individual should “follow their gut” when it tells them to “*Build Higher Levees*” than typically recommended by programs – such as the ***Logic & Statistics Program*** and the ***Heuristics & Bias Program*** - that use “*Rational*” decision-making tools, and models optimized from “*Mild*” randomness.

This chapter continues to illustrate how intuition leaps over the steps of “*Rational*” analysis with the human perception of what looks real based on its “*Roughness*”, and this takes us to ***Benoit Mandelbrot***.

## ***Benoit Mandelbrot***

***Benoit Mandelbrot***’s work focused on “*Discontinuity*” and “*Concentration*”. This focus led to a change in perspective from modeling “*Randomness*” as a measurement error, to modeling “*Randomness*” as a structural feature of the nature of reality. Using the perspective of ***Ole Peters***’ ***Ergodicity Economics***, “*Randomness*” becomes an intrinsic feature of a “*Growth Dynamic*” before it becomes a measurement error, or an appeal to a hidden force outside of the model. ***Mandelbrot’s*** work led him to create and develop a new field in mathematics based on a quantitative measure of “*Roughness*” in realized trajectories, either from evidence data, or from modeled simulations. He gave this field a name of his choosing: “*Fractals*”.

In 1963, ***Mandelbrot*** showed that scaling models of changes in asset prices based on the “*Axioms, Assumptions & Hypotheses*” of continuity, independence, and normality – such as ***Bachelier’s*** “*Random Walk*” with “*Brownian Motion*” - understate the nature and the level of price changes. From this point on, he developed models that do not depend on making such assumptions to show that price changes exhibit intermittent discontinuities (Jumps), and non-normality (Fat Tails).

Fractals describe mathematics objects and trajectories with irregular geometric shapes of dimension (D) with repeating structures of self-similarity at all scales (s) up to infinity, and quantified with a measure (L). The equation L = 1 / (s^D) articulates the quantitative relationship between these three variables. Fractals represent the invariant sets of iterated functions whose dimensions, unlike traditional geometric figures, expand beyond the integers to include real, irrational, as well as complex numbers.

Fractals’ self-similar structures at all scales do not have smooth and continuous trajectories, and instead exhibit rough and discontinuous behaviors. This means that we cannot calculate the rate of change [*The slopes, or the tangents of derivatives obtained through differentiation*] of fractal trajectories. Fractal dimensions provide comparable measures of changes in length, area, volume, or hyper-volume with changes in scale.

Finally, fractals connect with “*Chaos Theory*” through the process of iteration. Fractals show iterative convergence to self-similarity across all scales. Chaos shows iterative divergence through error propagation when the error grows to the same scale as the original signal.

In his 1997 book titled “*Fractals and Scaling in Finance*”, ***Mandelbrot*** articulates a spectrum of “*Randomness*”. This spectrum of “*Randomness*” has seven states that break into three categories as follows:

“*Mild Randomness*” – analogous to solids in Physics
“*Slow Randomness*” – analogous to liquids, and
“*Wild Randomness*” – analogous to gases.

The seven states of “*Randomness*” related to known probability distribution as follows:

(1) Proper “*Mild Randomness*”, such as the Normal Distribution,
(2) “*Borderline Mild Randomness*”, including specific Exponential Distributions,
(3) “*Delocalized Slow Randomness*”,
(4) “*Localized Slow Randomness*”, such as Lognormal Distributions,
(5) “*Pre-Wild Randomness*”, including specific Pareto Distributions,
(6) “*Wild Randomness*”, including specific Pareto Distributions, and
(7) “*Extreme Randomness*”, where all moments are infinite such as the Log-Cauchy Distribution.

As you can see from ***Mandelbrot***’s list of seven states of “*Randomness*”, the Lognormal Distribution, commonly assumed to provide a better model for the behavior of prices of risky assets than the Normal Distribution, takes us away from "*Mild Randomness*", and its traditional "*Axioms, Assumptions & Hypotheses*".

***Mandelbrot*** explains that financial reality shows "*Random Jumps*" rather than "*Random Walks*". This suggests the existence of material model risk in financial theories based on "*Mild Randomness*", such as Modern Portfolio Theory (MPT). ***Nassim Nicholas Taleb*** popularized the existence and the consequences of "*Wild Randomness*" in his books, ranging from "*Fooled by Randomness*" in 2002, to "*Statistical Consequences of Fat Tails*" in 2020. He branded rare events with outsized consequences – "*Black Swans*".

## ***Meder, et al.***

When ***David Meder*** presented the process and results of their Copenhagen Experiment at the EE2021 Conference, their use of fractals seemed odd at the time. He presented results from a 2019 paper titled "*Ergodicity-breaking reveals time optimal economic behavior in humans*", and his long list of co-authors included: ***David Meder, Finn Rabe, Tobias Morville, Kirstoffer H. Madsen, Magnus T. Koudahl, Ray J. Dolan, Hartwig R. Siebner, and Oliver J. Hulme***.

At the time, their use of fractals to teach the subjects "*via observation, the deterministic effect of fractal stimuli on their endowed wealth*" looked like adding confusing complexity to an already complex experiment. However, additional readings on the topic, such as the 2019 paper by ***Catherine Viengkham, Zoey Isherwood & Branka Spehar***, titled "*Fractal-Scaling Properties as Aesthetic Primitives in Vision and Touch*" suggest that our perception systems evolved to process fractals efficiently. At this time, ***Meder, et al.***'s use of fractals in an experiment that validated the premises of ***Ergodicity Economics*** looks like good common sense.

The fractal efficiency of our visual system makes it possible for our "*Brains*" to manage "*Motions*" through "*Predictions*" in a world filled with fractal structures – irregular geometric shapes that exhibit self-similarity across all scales. Thus, fractals have shapes "*made of parts similar to the whole*" according to ***Mandelbrot***. If you look around, you will see natural fractal structures everywhere, including clouds, mountains, and trees.

Interestingly, ***Viengkham, et al.*** (2019) suggests that these properties seem present in other types of perceptions, such as touch. Thus, it takes a short step to think that given the existence of fractal aesthetic primitives in vision and touch, our Preferences may also have a fractal geometry of their own. An additional data point comes from a paper written by ***Linfei Wu, Cheng-Jun Wang, Marco Jenssen, Jiang Zhang, and Min Zhao*** published in 2015, and titled: "*The Hidden Geometry of Attention Diffusion*". The paper notices that the topology of their ball-like analogy for attention diffusion makes an explicit connection to the quantification of "*Growth Dynamics*".

Attention flows like a river, its course shaped by changing preferences, especially in the presence of absorbing barriers. This leads to another short step to connect the Preference

Problem to fractals models for the diffusion of attention. Financial prices, Preferences about financial risk/returns reflect the diffusion of human attention. This means that the Problem of Preference leads to developing models that combine probability distributions other than those of smooth “Mild Randomness”, and with growth processes other than the simple linear and multiplicative versions.

***Richard Prum***

The structural limitations of “*Calculated Projections*” on lower dimensions presented in Chapter 4 apply to the Preference Problem as well as the Measurement Problem, but it does so with a twist. The quantification of the Preference Problem takes us beyond the smooth curves of Euclidian Geometry, and into the world of fractal geometry.

***Richard Prum*** connected individual subjectiveness with the requisite “*Randomness*” for proper “*Null Models*” in Hypothesis Testing. Differing types of “*Randomness*” in the evidence may come from the levels of dimension reduction that comes with subjectivity, and this dimensional reduction may come in the form of numbers on the real and imaginary number lines, instead of just the integer number line. “*Rational*” mental maps may be smooth, simple & continuous, but Real Life mental maps look rough, complex, and discontinuous. ***Prum*** wants the “*Null Model*” to reflect the nature of the evidence in the data, and the nature of the data about bird preferences looks fractal. One may even be able to assign a specific fractal dimension to the level of roughness and complexity of one’s own subjectivity.

This chapter connected the Preference Problem with “*Perceptions*” of roughness, complexity & randomness about growth [*Changes in “Motions”*], scale [*Changes in “Perspectives”*], and the existence of absorbing barriers ‘[*Changes in preferences and behaviors as one approaches event horizons*]. Human preferences come from the intuitive “*Perceptions*” of rough, complex & random “*Processes*”. Thus, other-than-mild “*randomness*” from such individual preferences should drive stochastic processes in models for decision-making, and “*Null Models*” for Hypothesis Testing in retirement planning.