Part IV – Section #11: Calculating Projections with the Tools of 19th Century (2 of 4)

Sections #10 through 14 present a selection of "*Calculated Projections*" with the tools of 19th Century. These "*Calculated Projections*" show the foundational presence of alternatives to "*Expected Value*", including "*Time Average*".

The third step of these "*Calculated Projections*" articulates the difference in calculating an arithmetic mean vs. a geometric mean as a decision criterion.

The arithmetic mean calculated in (Eq. IV.5.a) uses single period rates of return, r(a), converted into multipliers as shown in Part IV – Section #10, and equation (Eq. IV.5.b) converts it into an arithmetically averaged single period rate of return, r[i,a](t, t-1). This calculation uses an addition followed by a division, and a subtraction:

- (Eq. IV.5a): (1 + r[i,a](t, t-1)) = [(1 + r[i,s](t, t-1)) + (1 + r[i,s](t-1,t-2)) + …
  + (1 + r[i,s](t-n-1, t-n)] / n

- (Eq. IV.5b): r[i,a](t, t-1) = ([(1 + r[i,s](t, t-1)) + (1 + r[i,s](t-1,t-2)) + …
  + (1 + r[i,s](t-n-1, t-n)] / n) -1

The geometric mean calculated in (Eq. IV.6.a) uses a single period rate of return, r(g) converted into multipliers as shown in Part IV- Section #10, and equation (Eq. IV.6.b) converts it into a geometrically averaged single period rate of return, r[i,g](t, t-1). This calculation uses a multiplication, followed by a nth root calculation, and a subtraction:

- (Eq. IV.6a): (1 + r[i,g(t, t-1) = = [(1 + r[i,s](t, t-1)) * (1 + r[i,s](t-1,t-2)) * …
  * (1 + r[i,s](t-n-1, t-n)] ^(1/n)

- (Eq. IV.6b): r[i,g](t, t-1) = [(1 + r[i,s](t, t-1)) * (1 + r[i,s](t-1,t-2)) * …
  * (1 + r[i,s](t-n-1, t-n)] ^(1/n)] -1

Arithmetic means based on additions, divisions, and subtractions prove easier to calculate than geometric means based on multiplications, nth-roots, and subtractions. This alone may explain the dominant, historical popularity of "*Expected Value*" as a decision criterion. However, the commoditization of computing power in the 21st Century, and the awareness of its existential value in times of material change, may change the cost/benefit ratio in favor of "*Time Average*".

The fourth step articulates an important difference between rates of return and multipliers.

Adding 1 to (Eq. IV.1) - the single period rate of return equation - turns the rate of return percent, that can be a positive or a negative number, into a decimal multiplier that cannot be a negative number, but instead shows values greater or lesser than 1:

- (Eq. IV.1.b): ([x(t) – x(t-1)] / x(t-1)) + 1 = x(t)/x(t-1)

Thus, multipliers can also represent normalized prices on a trajectory when x(0) = 1.

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Multipliers of the 19th Century imply the "*Growth Dynamics*" of the 21st. Century. Decimals provide the same information as the percentage rate of return, but in a form that enables changing compounding frequencies, and (in the 20th Century) the use of calculus [The mathematics of Change based on the infinitely large & the infinitely small, as contrasted with the Mathematics of Chance based on discrete percentages & probabilities].

The arithmetic mean and the geometric mean return slightly different results when single period, simple rates of returns vary mildly from time-step to time-step, as shown in the table below.

					Arithmetic
Time	(t, t-1)	(t-1, t-2)	(t-2, t-3)	(t-3, t-4)	Mean
Rate	10.00%	8.00%	-4.00%	12.00%	6.50%
Multiplier	1.10	1.08	0.96	1.12	1.07
					Geometric
Time	(t, t-1)	(t-1, t-2)	(t-2, t-3)	(t-3, t-4)	Mean
Rate	10.00%	8.00%	-4.00%	12.00%	6.31%
Multiplier	1.10	1.08	0.96	1.12	1.06

The arithmetic mean and the geometric mean return very different results when single period, simple rates of returns vary wildly from time-step to time-step, as shown in the table below.

					Arithmetic
Time	(t, t-1)	(t-1, t-2)	(t-2, t-3)	(t-3, t-4)	Mean
Rate	50.00%	-60.00%	40.00%	-20.00%	2.50%
Multiplier	1.50	0.40	1.40	0.80	1.03
					Geometric
Time	(t, t-1)	(t-1, t-2)	(t-2, t-3)	(t-3, t-4)	Mean
Rate	50.00%	-60.00%	40.00%	-20.00%	-9.46%
Multiplier	1.50	0.40	1.40	0.80	0.91

This means that:

- "*Calculated Projections*" with the tools of the 19th Century of multi-period, average growth rates can show more than one answer depending upon the selected method of averaging, and
- Given sufficient levels of change in the single period, simple rates of returns from time step to time step, the arithmetic mean can show a positive average return, and the geometric mean can show a negative average return.

Thus, reasons for caution & skepticism about results from the ***Logic & Statistics Program*** extend from assumptions & hypotheses, as seen in Part II – Section #12, to models & methods. The "*Number Magic*" of Mathematics expressed by ***Richard Hamming*** in Part II – Section #6, similarly to the "*Word Magic*" of language expressed by ***C.K. Ogden & I.A. Richards*** in Part II – Section #3, does provide a "*Flexible Rule*".

Not only must we understand human relationships (e.g. "Who Benefits?) to make good individual decision, but even the basic use of mathematical results requires that we "*Observe*" & "*Orient*" ourselves explicitly about the assumptions, hypotheses, models & methods, theories, rules & laws behind such "*Decision*" tools for fear of "*Willful Ignorance*", Error & Deceit, especially so for decision tools used to nudge, censor, or mandate us into "*Action*".