"*Constructive Skepticism*" Volume 3 – Notebook #I: Model Risk

Chapter 4: "*Statistical Illusions*" from Statistical Machinery

These reading notes started with a review of the concept of "*Spinach*" [*Things we think unquestionably true but look ambiguously false after asking a few questions*] based on papers by ***Michael Sutton*** (Chapter 1) as well as a ***Michael Mielewczik & Jeanine Moll*** (Chapter 2) to develop a two-parts framework for the discovery of recurring cycles, and patterns of "*Willful Ignorance, Error & Deceit*" in research papers. Reading books by ***Rodolfo Llinas, Viktor Frankl & Richard Prum*** (Chapter 3) further developed the framework's (i) Measurement Problem ("*Statistical Meaning*"), and (ii) Preference Problem ("*Practical Meaning*") from first principles.

Chapter 3 closed with a focus on Hypothesis Testing as a "*Method*" to solve the Measurement Problem, and how ***Prum's*** "*Null Model*" Revolution meant choosing a proper model of "*Randomness*" as the "*Null Hypothesis*". This chapter starts with ***Francis Galton*** to show how the continued existence of "*Spinach*" about "*Randomness*" among researchers makes it hard for them to choose a proper "*Null Hypothesis*", and closes with ***Viktor Frankl*** to describe the model risk of all model risks: Confusing a part for the whole.

Francis Galton

The "*Puzzle of Regression to the Mean*" describes a statistical "*Effect*", a statistical illusion reminiscent of human optical illusions, that invites confusing the presence of a random subsequence of independent observations with an interpretation of causal consequences from the observations such that:

- Subsequent observations that follow extreme observations become more moderate, and
- The strength of this reversal, the "*Regression to the Mean*", comes from the weakness of the correlations between observations. [*A correlation assigns a number between -1 and +1 to describe how two variables change with respect to one another*].

Readers will benefit from making the distinction between the property of "*Independence of Observations*", and the property of "*Randomly Distributed Observations*".

- The property of "*Independence of Observations*" pertains to a relationship between two observations as measured by their level of correlation, and
- The property of "*Randomly Distributed Observations*" pertains to individual observations based on the "*Probability Distribution*" of their sample selection "*Process*" from the Population.

For instance, rolling a dice, and reading the top face of the dice will create a random list of numbers from a "*Theory*" that starts with a physical generation "*Process*" based on a uniform probability distribution with a probability of 1/6 per roll. With a "*Fair*" dice, one that does not have a "*Bias*", two successive observations will have a correlation of zero due to the Physics of this sample selection "*Process*", and thus show the property of "*Independence*".

The formula, Eq. 1.0: **$\langle r \rangle + \lambda\,(r(t) - \langle r \rangle) = r(t+1)$**, models the property of "*Independence*" such that the **r(t+1)** observation decomposes into the addition of the fixed value of the mean, **<r>**, and a variable value based on the prior observation minus the mean, times the level of correlation between observations.

In this example of rolling a dice, the property of "*Independence*" enables the "*Effect*" of the property of "*Randomly Distributed*" based on the Uniform Distribution so that "*Subsequent observations that follow extreme observations become more moderate*": If you roll a face with a 6, the probability of rolling a face other than a 6 in the next roll is greater than rolling another 6. This observation of reality through statistical machinery does not indicate a causal convergence to the dice's behavior toward its unrollable mean of [(1+2+3+4+5+6)/6 = 3.5].

Observing the "*Effect*" of "*Regression to the Mean*" means observing the combined properties of independent observations and random distribution. It does not mean observing a causal consequence from the observations. As shown with the mean of 3.5 in the dice example, this "*Effect*" does not create a causal convergence to a value that one can use as an "*Expected Value*" forecast for the next roll.

In a paper published by in "*The Journal of the Anthropological Institute of Great Britain and Ireland, Vol. 15 (1886) pp. 246-262*", and titled "*Regression Towards Mediocrity in Hereditary Stature*", ***Francis Galton***, FRS, &c. thought he had discovered a significant "*Effect*" from the "*Method*" of statistical averaging from sample data, Specifically, for a given range of parents' heights, the average height of the children appeared more moderate than the average height of the parents (including a 1.08 adjustment for women's heights). Galton called this effect "*Regression to the Mean*".

Galton confusion of the "*Effect*" of "*Randomness*" with an "*Effect*" from an assumed causal relationship illustrates the problem of inferring the existence of relationships with "*Methodologies"* based on "*As-if*" models that do not describe an underlying, causal model, but rely on teasing a statistical pattern out of the data. The same data can fit many patterns.

On the other hand, research based on "*Methodologies"* supported by a "*Theory*" benefit from having a "*Process*" model. A description of the model of genetic inheritance for measurable traits, such as height, would reveal a low correlation, and the presence of "*Randomness*". The presence of "*As-if*" models in a research paper triggers the red flag of high model risk, ***Galton*** et al.

This erroneous matching of "*Methodology*", "*Method*" and "*Interpretation*" continued in the 20th Century. ***Horace Secrist***'s book published in 1933, and titled: "*The Triumph of Mediocrity in Business*" provides another visible example because ***Harold Hotelling*** reacted to it by writing a paper titled "*Review of the Triumph of Mediocrity in Business, by Horace Secrist*". ***Hotelling*** corrected ***Secrist'*** error of confusing a regression analysis signal with random noise from imperfectly correlated variables.

This error of "*Interpretation*" continues to haunt research based on the statistical analysis of data in the 21st Century, as documented by ***Gary Smith*** in his 2016 paper, titled: "*A Fallacy that Will Not Die*". A reader involved in these Notebooks' public peer-review "*Process*" on Substack brought evidence of a similar problem in 2024, and raised the question: If researchers confuse the "*Statistical Meaning*" of "*Regression to the Mean*" from the presence of "*Randomness*" and the absence of correlations to the absence of "*Randomness*" and the presence of correlations, how can they choose the right "*Null Model*" for Hypothesis Testing?

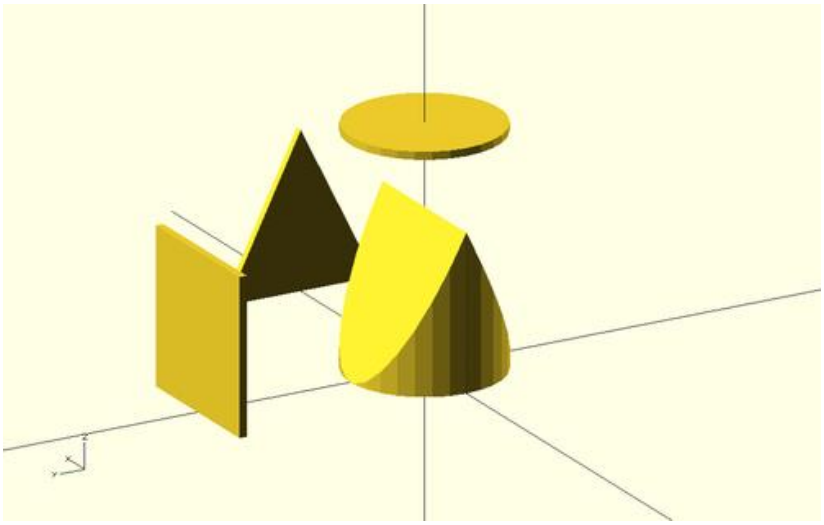
This brings us back to ***Viktor Frankl*** whom we first introduced in Chapter 3 to discuss the idea of "*Existential Meaning*" as a foundational drive for both "*Willful Ignorance, Error & Deceit*" in research, and for "*Two-in-One Mind*" thinking that can turn such a drive against itself in order to make valid inferences.

Viktor Frankl

In his 1969 book titled "*The Will to Meaning, Foundations and Applications of Logotherapy*" ***Viktor Frankl*** shows us why the model risk of all model risks comes from confusing a part for the whole. In the context of this examination of Model Risk, the risk of all risks comes from confusing "*Statistical Illusions*" with "*Statistical Significance*", and their authoritative "*Interpretation*" as "*Statistical Meaning*".

"*Axioms, Assumptions & Hypotheses*" as well as "*Models, Theories & Laws*" in various "*Domains of Knowledge*" summarize multi-dimensional evidence into "*Calculated Projections*" of lower dimensions. Thus, multi-dimensional evidence can have more than one legitimate "*Calculated Projection*" at lower dimensions, leaving us with seemingly contradictory findings. "*Calculated Projections*", such as "*Averages*" in general, and "*Expected Value*" in particular reduce dimensionality & complexity at the cost of a loss of information about the multi-dimensional evidence.

While ***Frankl*** used a 3-D Cylinder with two 2-D "*Calculated Projections*" to illustrate his point, this Notebook uses a 3-D physical object known as a Wedge-shaped Cylinder, as an example of multidimensional evidence, will summarize as three different 2-D "*Calculated Projections*". The projected views, from each of its up/down, left/right, and front/back perspectives, look like a Circle, a Triangle, and a Square.



Source: https://kitwallace.tumblr.com/post/103975175234/george-harts-circle-triangle-square-puzzle

Note the parallel between this picture and CTRI's name: The Curve, Triangle & Rectangle Institute.

- The Curve refers to Modern Portfolio Theory (MPT), and its foundations in the ***Logic & Statistics Program***.
- The Triangle refers to Behavioral Economics & Behavioral Finance, and their foundations in the ***Heuristics & Bias Program***.
- Finally, the Rectangle (the Square in the picture) refers to the Household Balance-Sheet, and its foundations in ***Ergodicity Economics*** and the "*Fast & Frugal*" ***Heuristics Program***.

These three different lower dimensional projections in the picture shown above explain why the Template for Reading Research Papers starts with the analytical step of "*Perspective*". We see what we understand. Models reduce reality to a map, and research papers will have different "*Perspectives*", and show different maps based on the "*Methodologies, Methods & Analogies*" that they use.

Frankl called this mapping "*Dimensional Ontology*" – the analysis of the various "*Calculated Projections*" that multi-dimensional evidence can take subject to reductions to lower dimensions, and identified two "*Laws*".

- ***Frankl's*** first "*Law*" of "*Dimensional Ontology*" states that "*One and the same phenomenon projected out of its own dimension into different dimension lower than its own is depicted in such a way that the individual pictures contradict one another*". We can see this in the picture shown above that the multi-dimensional evidence illustrated by the 3-D Wedge-shaped Cylinder creates three 2-D pictures that contradict one another: A Circle, a Triangle, and a Square.

- ***Frankl's*** second "*Law*" of "*Dimensional Ontology*" states that "*Different phenomena projected out of their own dimension into one dimension lower than their own are depicted in such manner that the pictures are ambiguous*." Again, we can infer this from the picture shown above: Akin to the 3-D Wedge-shaped cylinder, a 3-D Wedge-shaped Triangle would create a 2-D Triangle, as well as a 2-D Square in two of the three projections.

These illustrations suggest that we should remain skeptical of all models and their "*Calculated Projections*" because they cannot match the full dimensionality of the evidence. Statistical research that calls itself evidence-based claims a higher dimensionality that it can achieve.

For instance, ***Frankl*** asks us to reflect on the meaning of understanding that the Wedge-shaped Cylinder could have an open shape like a cup, instead of a closed shape like a solid. Echoing ***Hannah Arendt's*** point about "*Two-in-One Mind*" thinking, he uses this open shape to illustrate the open-ended quality of being human, and to show that the 2-D "*Calculated Projections*" would miss this foundational aspect of the multi-dimensional evidence.

Frankl saw these pictures as an explanation of the destruction created by scientific "*Reductionism*" where the ends justify the means. He also saw these pictures as an explanation of the destructive power of ideology from its ability to mandate an arbitrary 2-D projection on other people as the one-and-only truth-statement about the nature of the 3-D Wedge-shaped Cylinder. His observations apply to "*Models, Theories & Laws*" in all "*Domains of Knowledge*", including Financial Economics, and Retirement Planning.

Solving the Measurement Problem requires many lower-dimension simplifications in order to fit the mathematics as shown in the table below. While the development of this table came from research paper in the field of Financial Economics, its general outline comes from the mathematics of statistical machniery that apply to other "*Domains of Knowledge*".

Table of "*Axioms, Assumption & Hypotheses*" for Financial Economics Models

Axioms, Assumptions & Hypotheses
"Small Worlds" Equilibrium Task Environment
Random Variables
Continuous Change
Statistical Independence of Observables
Stationarity of the Change Process
Identically & Independently Distributed (IID) Random Variables
Normally Distributed Random Variables
Frictionless Environment
"Rational" Decision-Maker
Homo economicus
Bias-free & Rational
Utility-maximizer
Unlimited Cognitive Capacity
Narrow Self-Interest
Perfect Access to Information
Preference Consistency
Homogeneous Expectations
Game-theoretic Decision-Making Process
Continuity of Choice Combinations
Monotonicity (Completeness of Preferences & Choices)
Transitivity (Consistency of Choices)
Independence of Irrelevant Alternatives (IIA)
Calculations Short-Cuts
Dropping the Temporal Sub-script
Variance of Averages of RVs
Variance of Sums of RVs
Estimate of Population Mean

However, these simplifications quickly become complicated enough to make us miss the forest for the tree, and forget that the "*Calculated Projections*" create structurally lower dimension representations of the evidence, as we can see by describing the first item in the table: "*Random Variables*".

- A "*Random Variable*" provides a mathematical function that associates a number with "*Observables*", such that it has a probability distribution, the list of its numbered values matched to a probability between zero and one.
- In the case of continuous Random Variables, the probability distribution becomes a probability density function (PDF), and combining the PDFs of two Random Variables makes them jointly distributed.

"*Statistical Independence*", the third item on the list, means that in the case of jointly distributed "*Random Variables*", all of the necessary information about each variable can be obtained from the joint distribution.

- The alone distribution for each of the jointly distributed "*Random Variables*" is called the marginal distribution, and two "*Random Variables*" are statistically independent if their joint distribution equals the product of their marginal distributions.
- This means that compound probabilities will calculate as products of individual probabilities, and their logarithms convert the calculations into sums.

"*Statistical Independence*" means that a realization for one "*Random Variable*" does not affect the probability distribution of another "*Random Variable*", it implies zero correlation.

- This assumption of independence enables the use of statistical tools such as the Central Limit Theorem) [*A set of weak-convergence theorems in probability theory such that the Sum of normalized, independent random variables approximates a normal distribution, even when they are not originally normally distributed*], and
- The Law of Large Numbers [*A theorem in probability theory such that the average from a large number of trials should approximate the "Expected Value" of the population, and should become closer to it with additional trials*].

However, "*Large World*" data rarely shows "*Statistical Independence*", and frequently shows correlations, as well as statistical dependence over various time-frames, creating an important mismatch between models and empirical experience, thus opening up questions of "*Constructive Skepticism*" about the applicability of recommendations based on such models. Finally, additional simplifications extend beyond foundational "*Axioms, Assumptions & Hypotheses*" to include the use of short-cuts in the performance of financial calculations, such as the temporal sub-script simplification.

This table reinforces ***Frankl's*** point that solving the Measurement Problem with models and "*Calculated Projections*" brings multi-dimensional evidence down to lower dimensions, and that this will create statistical doubt. Further, the illustration with the geometric shapes shows that such models will likely contradict one another. Examples of other statistical illusions abound, and include Simpson's Paradox, as well as the use of Expected Values of Periodic Returns instead of Time Average Growth Rates with non-Ergodic Growth Dynamics.

As we close this chapter focused on the "*Statistical Illusions*" of the Measurement Problem, and prepare to move to the next chapter that will focus on the runaway trajectories of the Preference Problem, we can summarize the dots that we connected between ***Rodolfo Llinás, Viktor Frankl, Richard Prum, & Francis Galton*** as follows:

- Our evolutionary adaptation to make intelligent inferences in the presence of incomplete and uncertain information suggests that "*Brains*" developed to manage "*Motions*" through "*Predictions*".
- We use conceptual representations, "*Mind-Maps*" based on perceived "*Cause & Effect*", to summarize the "*Territory*" of our uncertain "*Task Environments*".

- These summaries, and "Calculated Projections" come as the cost of a loss of information, up to the point of unresolvable statistical doubt and clinical ambiguity.
- However, our objective experience of Nature "*Red in Tooth & Claw*" – The Measurement Problem - keeps our "*Predictions*" and "*Motions*" in check.
- On the other hand, the subjective experience of our own "*Mind-Maps*" has no such constraints on "*Mind Exuberance".*
- "*Primary Emotions*" – The Preference Problem - emerged to prioritize "*Observations*" based on "*Perceptions*", at the risk of runaway "*Bubble Formation*" in the "*Mind*".
- *Thus,* subjective "*Ideas, Beliefs & Ideologies*" as well as "*Models, Theories & Laws*" can go into "*Bubble Formation*", and over-reach past their range of usefulness.